LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

FOURTH YEAR EXAMINATION FOR THE DEGREE OF

BACHELOR OF EDUCATION (ARTS/SCIENCE) AND BACHELOR OF SCIENCE (GENERAL)

MATH 426: ORDINARY DIFFERENTIAL EQUATIONS II

STREAM: R ***TIME: 2 HRS***

DAY: WEDNESDAY [8.30 – 10.30 A.M] ***DATE: 17/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: Answer Question 1 and Any Other 2 Questions

QUESTION ONE (30 MARKS)

(a). By using reduction of order method solve the differential equation $y^2(y'') - (y')^3 = 0$

(5 Marks)

(b). Find the general solution of the following systems

(i). $$\frac{dy}{dt} = -6x + 2y$$
$$\frac{dx}{dt} = -3x + y$$
(5 Marks)

(ii) $$\frac{dy}{dt} = -6x + 5y$$
$$\frac{dx}{dt} = -5x + 4y$$
(5 Marks)

(iii). $$\frac{dy}{dt} = 5x + y$$
$$\frac{dx}{dt} = -2x + 3y$$
(5 Marks)

(c). Find the radius of convergence and interval of convergence of the given power series $\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$

(5 Marks)

(d). Determine the singular points of the equation $(x^3 + 4x)y'' - 2xy' + 6y = 0$, hence classify each singular point as regular or irregular **(5 Marks)**

QUESTION TWO (20 MARKS)

(a). Verify that the vector X is a solution of the given system $X' = \begin{pmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{pmatrix} X$, $X = \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-\frac{3}{2}t}$

(4 Marks)

(b). Use the method of variation of parameters to solve the homogeneous system $X' = Ax + F(t)$, given as $X' = \begin{pmatrix} 1 & 8 \\ 1 & -1 \end{pmatrix} X + \begin{pmatrix} e^{-t} \\ te^{t} \end{pmatrix}$ **(10 Marks)**

(c). solve the homogeneous linear system $\begin{aligned} \frac{dx_1}{dt} &= 4x_1 + 3x_2 + x_3 \\ \frac{dx_2}{dt} &= -4x_1 - 4x_2 - 2x_3 \\ \frac{dx_3}{dt} &= 8x_1 + 12x_2 + 6x_3 \end{aligned}$ **(6 Marks)**

QUESTION THREE (20 MARKS)

(a). (i).Show that $x = 0$ is an ordinary point for the equation $(x^2 - 1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0$ **(2 Marks)**

(ii). Find the power series solution for the equation given in question 3(a). **(10 Marks)**

(b). Find all the critical points of the nonlinear system $\begin{aligned} \frac{dx}{dt} &= 8x - y^2 \\ \frac{dy}{dt} &= -6y + 6x^2 \end{aligned}$ and determine the type and stability of each of the critical points. **(8 Marks)**

QUESTION FOUR (20 MARKS)

(a). Use the method of undetermined coefficients to solve the system of differential equations

$$X' = \begin{pmatrix} 4 & \frac{1}{3} \\ 9 & 6 \end{pmatrix} X + \begin{pmatrix} -3 \\ 10 \end{pmatrix} e^t$$ **(10 Marks)**

(b). Use the Frobenius method to solve $2x^2 y'' - xy' + (1+x)y = 0$ **(10 Marks)**

QUESTION FIVE (20 MARKS)

(a). Solve the given initial value problem $X' = \begin{pmatrix} 1 & -12 & -14 \\ 1 & 2 & -3 \\ 1 & 1 & -2 \end{pmatrix} X, X(0) = \begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix}$ **(10 Marks)**

(b). Find all the critical points and discuss the qualitative behavior of the nonlinear autonomous system

$$\frac{dx}{dt} = -x + y^2$$
$$\frac{dy}{dt} = -y + x^2$$

(10 Marks)