LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STAT 121: INTRODUCTION TO PROBABILITY AND STATISTICS II

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY [8.30A.M -10.30A.M] ***DATE: 19/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: Answer **QUESTION ONE** and any other **TWO** questions

QUESTION ONE (30 MARKS)

a) The continuous random variable X has probability density function

$$f(x) = \begin{cases} kx^2, & 0 < x < 2 \\ 0 & elsewhere \end{cases}$$, where k is a constant.

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i) What must be the value of k? **(2 Marks)**

ii) Find the cumulative distribution function (CDF), F_X, of X and sketch its graph. **(5 Marks)**

iii) Calculate $P(1 < X < 2)$ **(2 Marks)**

b) Suppose that the number of customers that arrive at a travel agency is assumed to follow a Poisson distribution with a mean of 15 customers per hour. Find the probability that

i) Exactly 10 customers arrive in 1-hour period

ii) At least 20 but less than 23 customers arrive in a 2-hour period. **(5 Marks)**

c) Suppose that for a certain company the number of automobiles that are used for official business purposes on any given workday is a random variable X with the following probability mass function:

x	1	2	3	4
$f(x)$	0.10	0.45	0.15	0.30

i) Find the mean and standard deviation of the number of automobiles used for official business purposes on a given workday.

ii) Find the probability that the number of automobiles used for official business purposes on a given workday will be less than 3. **(6 Marks)**

d) Let the random variable X have the MGF given by the expression

$M_X(t) = (0.2 + 0.8e^t)^{20}$, find the probability that X is greater than 13 but less than or equal to 17. **(5 Marks)**

e) Each customer entering a certain store will make a purchase with probability 0.4. What is the probability that, among 1000 customers exactly 423 make purchases. **(5 Marks)**

QUESTION TWO (20 MARKS)

a) Suppose that X is a random variable for which $E(X) = 8$ and $Var(X) = 12$. Find

i) $E[(2+X)^2]$

ii) $Var(5X-4)$ **(6 Marks)**

b) Products produced by a machine has a 4% defective rate. What is the probability that the first defective occurs in the sixth item inspected? **(4 Marks)**

c) The probability that a person recovers from a rare blood disease is 0.4. Suppose 15 people are known to have contracted this disease. Find the probability that

i) Exactly five recover

ii) At least four but not more than six recover? **(5 Marks)**

d) A student prepares for an exam by studying a list of 10 problems. She can solve six of them. For the exam, the instructor selects five questions at random from the list of ten. What is the probability that the student can solve exactly three problems on the exam? **(5 Marks)**

QUESTION THREE (20 MARKS)

a) The speeds of cars are measured using a radar unit, on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 12 km/hr. Find the probability that a randomly selected car is moving at

i) Less than 100 km/hr

(**4 Marks**)

ii) More than 85 km/r

(**4 Marks**)

iii) Between 96 and 112 km/hr

(**6 Marks**)

b) The managers of an electronics firm estimate that 70% of the new products they market will be successful. If the company markets 100 products over the next five years, find the probability that more than 85 new products will be successful. **(6 Marks)**

QUESTION FOUR (20 MARKS)

a) A rifleman fires at a target 3 times and the probability is 0.7 that he hits the target each time. Let X denote the number of times he hits the target. Find the probability distribution, mean and variance of the random variable X. **(7 Marks)**

b) The owner of a new apartment building must install 15 water heaters. The length of life a certain brand is a random variable having an exponential distribution with a mean of 8 years. What is the probability that between 5 and 7 of the hot water heaters will last at least 10 years? **(7 Marks)**

a) A random variable X has its moment generating function given by $M(t) = \frac{1}{5}\left(e^{t} + 2e^{4t} + 2e^{8t}\right)$ for $-\infty < t < \infty$,

a) Find μ and σ^2

b) Identify the probability mass function of X **(6 Marks)**

QUESTION FIVE (20 MARKS)

a) If a random variable X is defined such that $E\left[(X-1)^2\right] = 10$ and $E\left[(X-2)^2\right] = 6$.

Find the mean and variance of X. **(5 Marks)**

b) Let X be a continuous variable with density function $f(x) = \begin{cases} ax + bx^2, & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$. If $E(X) = 0.6$, find

i) The values of a and b.

ii) $P\left(X > \frac{1}{2}\right)$

iii) $Var(X)$ **(10 Marks)**

c) If X is a random variable such that $E(X) = 5$ and $E(X^2) = 29$, use Chebyshev's inequality to determine a lower bound for the probability $P(-2 < X < 12)$ **(5 Marks)**